

SRI VENKATESWARA UNIVERSITY - TIRUPATI
B.Sc., (Honours) in MATHEMATICS (MINOR)
V SEMESTER
(W.E.F. Academic Year 2025 - 26)

COURSE 12: LINEAR ALGEBRA

Theory

Credits: 4

5 hrs. / week

Course Outcomes

After successful completion of this course, the student's will be able to

1. Understand the concepts of vector spaces, subspaces.
2. Understand the concepts of basis, dimension and their properties.
3. Understand the concept of linear transformation and its properties.
4. Apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods.
5. Learn the properties of inner product spaces and determine orthogonality in inner product spaces.

Course Content

UNIT – I

Vector Spaces-I

Vector Spaces - General properties of vector spaces - n-dimensional Vectors - addition and scalar multiplication of Vectors - internal and external composition - Null space - Vector subspaces - Algebra of subspaces - Linear Sum of two subspaces - linear combination of Vectors- Linear span Linear independence and Linear dependence of Vectors.

UNIT –II

Vector Spaces-II

Basis of Vector space - Finite dimensional Vector spaces - basis extension - coordinates- Dimension of a Vector space - Dimension of a subspace - Quotient space and Dimension of Quotient space.

UNIT –III

Linear Transformations

Linear transformations - linear operators- Properties of L. T- sum and product of L. Ts - Algebra of Linear Operators - Range and null space of linear transformation - Rank and Nullity of linear transformations - Rank- Nullity Theorem.

UNIT –IV

Matrices

Characteristic equation - Characteristic Values - Characteristic vectors of a square matrix - Cayley Hamilton Theorem – problems on Cayley Hamilton Theorem.

UNIT –V

Inner product space

Inner product spaces- Euclidean and unitary spaces- Norm or length of a Vector- Schwartz inequality- Triangle Inequality- Parallelogram law- Orthogonality- Orthonormal set - Bessel's inequality.

Activities:

Seminar / Quiz / Assignments / Applications of Linear Algebra in real life problems / Problem Solving.

Text Books

1. Linear Algebra by J.N. Sharma and A.R. Vasishtha, published by Krishna Prakashan Media (P) Ltd.
2. Matrices by A.R. Vasishtha and A.K. Vasishtha, published by Krishna Prakashan Media (P) Ltd.

Reference Books

1. Linear Algebra by Stephen H. Friedberg et. al. published by Prentice Hall of India Pvt. Ltd. 4th Edition, 2007.
2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson education , New Delhi.
3. Matrices by Shanti Narayana, published by S. Chand Publications.

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S.V.UNIVERSITY, TIRUPATI
MODEL PAPER
THREE YEAR B.A/B.Sc. DEGREE EXAMINATIONS
V SEMESTER
Subject: MATHEMATICS
Paper: LINEAR ALGEBRA

Time: 3 hours

Max Marks: 75

SECTION – A

Answer any FIVE of the following questions.

5X5 = 25 M

1. Show that the set $W = \{ (x - y, y - z, 0) : x, y, z \in R \}$ is a subspace of R^3 .
2. Verify that the polynomials $1 + 2t^2, 4 + t + 5t^2$ and $3 + 2t$ are linearly independent in P_2 .
3. Find the basis and the dimension of the subspace $H = \{ (s - 2t, s + t, 3t) : s, t \in R \}$ of the vector space R^3 .
4. Let $T : R^2 \rightarrow R^2$ be a linear transform with $T(e_1) = (3, 1, 3, 1)$ and $T(e_2) = (-5, 2, 0, 0)$, where $e_1 = (1, 0)$ and $e_2 = (0, 1)$. Then find T.
5. Describe explicitly the linear transformation $T : R^2 \rightarrow R^2$ such that $T(2, 3) = (4, 5)$ and $T(1, 0) = (0, 0)$.
6. Let $T : V \rightarrow W$ be a linear transformation. Then prove that Range of T is a subspace of W.
7. Find the eigen value of the matrix
$$\begin{pmatrix} 7 & 0 & 0 \\ 8 & -4 & 0 \\ 1 & 5 & 2 \end{pmatrix}$$
.
8. State and prove parallelogram law in Inner product space.

SECTION – B

Answer ALL of the following questions.

5X10 =50 M

9. a) Prove that the set of the polynomials with real coefficients of degree at most n is a vector space over real numbers.

OR

b) Let W be a non-empty subset of a vector space $V(F)$. Then prove that W is a subspace of V if $a\alpha + b\beta \in W \forall a, b \in F$ and $\alpha, \beta \in W$.

10. a) Let $V(F)$ be a finite dimensional vector space of dimension n and W be a subspace of V . Then prove that W is a finite dimensional and $\dim W \leq n$.

OR

b) Let W_1 and W_2 be two subspaces of vector space R^4 given by

$$W_1 = \{(a, b, c, d) : b - 2c + d = 0\} \text{ and } W_2 = \{(a, b, c, d) : a = d, b = 2c\}.$$

Then find $\dim(W_1 + W_2)$.

11. a) State and prove Rank – Nullity theorem.

OR

b) Find the range and null space of a linear operator on R^3 defined by

$$(a, b, c) \mapsto (3a, a - b, 2a + b + c).$$

12. a) State and prove Cayley - Hamilton theorem.

OR

b) Prove that the eigen vectors of a distinct eigen values are linearly independent.

13. a) State and prove Schwartz Inequality.

OR

b) State and prove Bessel's Inequality.

SRI VENKATESWARA UNIVERSITY - TIRUPATI
B.Sc., (Honours) in MATHEMATICS (MINOR) V
SEMESTER
(W.E.F. Academic Year 2025 - 26)

COURSE 13: VECTOR CALCULUS

Theory

Credits: 4

5 hrs. / week

Course Outcomes

After successful completion of this course, the student's will be able to

1. Learn multiple integrals as a natural extension of definite integral to a function of two variables in the case of double integral/three variables in the case of triple integral.
2. Learn applications in terms of finding surface area by double integral and volume by triple integral.
3. Determine the gradient, divergence and curl of a vector and vector identities.
4. Evaluate line, surface and volume integrals.
5. Understand relation between surface and volume integrals (Gauss divergence theorem), relation between line integral and volume integral (Green's theorem), relation between line and surface integral (Stokes theorem)

Course Content

Unit – I

Multiple integrals I

Introduction – Double integrals – Evaluation of double integrals – Properties of double integrals – Region of integration – Double integration in polar coordinates – Change of variables in double integrals – change of order of integration.

Unit-2

Multiple integrals II

Triple integral – region of integration – change of variables – Plane areas by double integrals - surface area by double integral – Volume as a triple integral.

Unit-3

Vector differentiation

Vector differentiation –ordinary – derivatives of vectors – Differentiability –Gradient – Divergence – Curl operators – Vector Identities.

Unit-4

Vector integration

Line Integrals with examples - Surface Integral with examples – Volume integral with examples.

Unit-5

Vector integration applications

Gauss theorem and applications of Gauss theorem - Green's theorem in a plane and applications of Green's theorem - Stokes's theorem and applications of Stokes theorem.

Activities

Seminar / Quiz / Assignments / Applications of Vector calculus to Real life Problems / Problem Solving Sessions.

Text Book

A text Book of Higher Engineering Mathematics by B.S. Grewal, Khanna Publishers, 43rd Edition

Reference Books

1. Vector Calculus by P.C. Matthews, Springer Verlag publications.
2. Vector Analysis by Murray Spiegel, Schaum Publishing Company, New York

S.V.UNIVERSITY, MODEL PAPER
THREE YEAR B.A/B.Sc. DEGREE EXAMINATIONS
V SEMESTER
Subject: MATHEMATICS
Paper: VECTOR CALCULUS

Time: 3 hours

Max Marks:75

SECTION – A

Answer any FIVE of the following questions.

5X5 = 25 M

1. Evaluate $\int_0^4 \int_0^2 e^{y/x} dx dy$.
2. Evaluate $\int_{-1}^0 \int_0^1 \int_{x-z}^{+z} (x + y + z) dz dx dy$.
3. Evaluate $\iiint x y z dx dy dz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming into spherical co-ordinates.
4. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$, $x^2 + y^2 + z = 3$ at $(2, -1, 2)$.
5. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ where C is the curve $y = 2x^2$ in the xy-plane.
6. Evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where 'C' is circle $x^2 + y^2 = 1, z = 0$ and $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$.
7. Verify green's theorem in the plane $\oint_C (x + y + y^2)x + x^2 dy$. Where 'C' is closed curve of the region bounded by $y = x$, & $y = x^2$.
8. Show that $\text{grad } r^m = m r^{m-2} \cdot \vec{r}$

SECTION – B

Answer ALL of the following questions. Each question carries 10 marks (5X10 =50).

9. a) Evaluate $\int_0^a \int_{y^2}^{\sqrt{a^2 - x^2}} \sqrt{a^2 - x^2 - y^2} dx dy$.

OR

b) Evaluate $\iint (x + y)^2 dx dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

10. a) Using double integral find area enclosed by the curves $y = 2x^2$ and $y^2 = 4x$.

OR

b) Find the area of the surface of the sphere $x^2 + y^2 + z^2 = 9a^2$ cutoff by the cylinder

$$x^2 + y^2 = 3ax.$$

11. a) If $r = a \cos t \mathbf{i} + a \sin t \mathbf{j} + at \tan \theta \mathbf{k}$ then find $\left| \frac{dr}{dt} \right|^2$ and $\left[\frac{r}{dt} \cdot \frac{d^2 r}{dt^2} \cdot \frac{d^3 r}{dt^3} \right]$.

OR

b) If $\vec{F} = \text{grad} (x^3 + y^3 + z^3 - 3xyz)$ then find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$.

12. a) Evaluate $\int_S \vec{F} \cdot \vec{N} \, ds$, $\vec{F} = z\mathbf{i} + x^2\mathbf{j} - 3y^2\mathbf{k}$ and 'S' is the surface $x^2 + y^2 = 16$ included in first octant between $z=0$ & $z=5$.

OR

b) If $F = 2xz \mathbf{i} - x \mathbf{j} + y^2 \mathbf{k}$ evaluate $\int_V F \, dv$ where V is the region bounded by the surface $x=0$, $x=2$, $y=0$, $y=6$, $z=x^2$, $z=4$.

13. a) State and prove Gauss's divergence theorem.

OR

b) State and prove Stoke's theorem.