SRI VENKATESWARA UNIVERSITY

B.A / B.Sc. DEGREE COURSE IN MATHEMATICS

SEMESTER SYSTEM WITH CBCS

SEMESTER IV - W.E.F. 2021-2022

COURSE IV: REAL ANALYSIS

(75 Hours)

Course Outcomes:

After successful completion of this course, the student will be able to

1. get clear idea about the real numbers and real valued functions.
2. obtain the skills of analyzing the concepts and applying appropriate methods for testing convergence of a sequence/ series.
3. test the continuity and differentiability and Riemann integration of a function.
4. know the geometrical interpretation of mean value theorems.

Course Syllabus:

UNIT – I (12 Hours)

REAL NUMBERS :

The algebraic and order properties of R, Absolute value and Real line, Completeness property of R, Applications of supremum property; intervals. (No question is to be set from this portion).

REAL SEQUENCES:

Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence. The Cauchy's criterion, properly divergent sequences, Monotone sequences, Necessary and Sufficient condition for Convergence of Monotone Sequence, Limit Point of Sequence, Subsequences and the Bolzano-weierstrass theorem – Cauchy Sequences – Cauchy's general principle of convergence theorem.

UNIT –II (12 Hours)

INFINITIE SERIES :

SERIES : Introduction to series, convergence of series. Cauchy's general principle of convergence for series tests for convergence of series, Series of Non-Negative Terms.

1. P-test
2. Cauchy's n^{th} root test or Root Test.
3. D'-Alemberts' Test or Ratio Test.
4. Alternating Series – Leibnitz Test.

Absolute convergence and conditional convergence.

UNIT – III (12 Hours)

CONTINUITY :

LIMITS : Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limit concept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).

Continuous functions : Continuous functions, Combinations of continuous functions, Continuous Functions on intervals, uniform continuity.

UNIT – IV (12 Hours)

DIFFERENTIATION AND MEAN VALUE THEORMS :

The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean value Theorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

UNIT – V (12 Hours)

RIEMANN INTEGRATION :

Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Properties of integrable functions, Fundamental theorem of integral calculus, integral as the limit of a sum, Mean value Theorems.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

Text Book:

Introduction to Real Analysis by Robert G.Bartle and Donlad R.Sherbert, published by John Wiley.

Reference Books:

1. A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand & Company Pvt. Ltd., New Delhi.
2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania, published by S. Chand & Company Pvt. Ltd., New Delhi.

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SEMESTER IV - W.E.F. 2021-2022

COURSE-IV REAL ANALYSIS

Time: 3Hrs **Max.Marks:75M**

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1.Prove that every convergent sequence is bounded.

2. Show that $\lim\left(\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\text{------------------}+\frac{1}{(n+n)^2}\right)=0$.

3. Test the convergence of the series $\sum_{n=1}^{\infty}(\sqrt[3]{n^3+1}-n)$.

4. Examine for continuity of the function f defined by $f(x)=|x|+|x-1|$ at x=0 and 1.

5. Show that $f(x)=x\sin\frac{1}{x},\ x\neq 0; f(x)=0, x=0$ is continuous but not derivable at x=0.

6.Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2+11x-6$ on $[1,3]$.

7. If $f(x)=x^2\ \forall\, x\in[0,1]$ and $p=\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\}$ then find L(p, f) and U(p, f).

8.prove that if $f:[a,b]\to R$ is continuous on [a, b] then f is R- integrable on [a, b].

SECTION –B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9.(a)If $s_n=1+\frac{1}{2!}+\frac{1}{3!}+------+\frac{1}{n!}$ then show that $\{s_n\}$ converges.

(OR)

(b) State and prove Cauchy's general principle of convergence.

10. (a) State and Prove Cauchy's nth root test.

(OR)

(b) Test the convergence of $\sum \frac{x^n}{x^n+a^n}$ $(x > 0, a > 0)$.

11.(a) Let f: R→ R be such that

$$f(x) = \frac{\sin(a+1)x+\sin x}{x} \quad \text{for } x < 0$$

$$= c \qquad \text{for } x = 0$$

$$= \frac{(x+bx^2)^{1/2}-x^{1/2}}{bx^{3/2}} \quad \text{for } x > 0$$

Determine the values of a, b, c for which the function f is continuous at x=0.

(OR)

(b) Define uniform continuity, If a function f is continuous on [a b] then f is uniformly continuous on [a b]

12.(a) Using Lagrange's theorem, show that

$$x > \log(1+x) > \frac{x}{(1+x)} \, \forall \, x > 0$$

. (OR)

(b) State and prove Cauchy's mean value theorem.

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13.(a) State and prove Riemman's necessary and sufficient condition for R- integrability.

(OR)

(b) Prove that $\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \le \frac{\pi^3}{6}$.

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SEMESTER SYSTEM WITH CBCS

SEMESTER IV - W.E.F. 2021-2022

COURSE: V LINEAR ALGEBRA

(75 Hours)

Course Outcomes:

After successful completion of this course, the student will be able to;

1. understand the concepts of vector spaces, subspaces, basises, dimension and their properties
2. understand the concepts of linear transformations and their properties
3. apply Cayley- Hamilton theorem to problems for finding the inverse of a matrix and higher powers of matrices without using routine methods
4. learn the properties of inner product spaces and determine orthogonality in inner product spaces.

Course Syllabus:

UNIT – I (12 Hours)

Vector Spaces-I:

Vector Spaces, General properties of vector spaces, n-dimensional Vectors, addition and scalar multiplication of Vectors, internal and external composition, Null space, Vector subspaces, Algebra of subspaces, Linear Sum of two subspaces, linear combination of Vectors, Linear span Linear independence and Linear dependence of Vectors.

UNIT –II (12 Hours)

Vector Spaces-II:

Basis of Vector space, Finite dimensional Vector spaces, basis extension, co-ordinates, Dimension of a Vector space, Dimension of a subspace, Quotient space and Dimension of Quotient space.

UNIT –III (12 Hours)

Linear Transformations:

Linear transformations, linear operators, Properties of L.T, sum and product of LTs, Algebra of Linear Operators, Range and null space of linear transformation, Rank and Nullity of linear transformations – Rank – Nullity Theorem.

UNIT –IV (12 Hours)

Matrix :

Matrices, Elementary Properties of Matrices, Inverse Matrices, Rank of Matrix, Linear Equations, Characteristic equations, Characteristic Values & Vectors of square matrix, Cayley – Hamilton Theorem.

UNIT –V (12 Hours)

Inner product space :

Inner product spaces, Euclidean and unitary spaces, Norm or length of a Vector, Schwartz inequality, Triangle Inequality, Parallelogram law, Orthogonality, Orthonormal set, complete orthonormal set, Gram – Schmidt orthogonalisation process. Bessel's inequality and Parseval's Identity.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Linear algebra and its applications / Problem Solving.

Text Book:

Linear Algebra by J.N. Sharma and A.R. Vasista, published by Krishna Prakashan Mandir, Meerut- 250002.

Reference Books :

1. Matrices by Shanti Narayana, published by S.Chand Publications.
2. Linear Algebra by Kenneth Hoffman and Ray Kunze, published by Pearson Education (low priced edition),New Delhi.
3. Linear Algebra by Stephen H. Friedberg et. al. published by

Prentice Hall of India Pvt. Ltd. 4th Edition, 2007.

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SEMESTER SYSTEM WITH CBCS

SEMESTER IV - W.E.F. 2021-2022

COURSE-V, LINEAR ALGEBRA

Time: 3Hrs

Max.Marks:75M

SECTION - A

Answer any FIVE questions. Each question carries FIVE marks 5 X 5 M=25 M

1. Let p, q, r be fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F, such that px+qy+rz=0 is a vector subspace of $V_3(R)$.

2. Define linearly independent &linearly dependent vectors in a vector space. If α, β, γ are linearly independent vectors of $V(R)$ then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ are also linearly indepen

3. Prove that every set of $(n+1)$ or more vectors in an n dimensional vector space is linearly dependent.

4. The mapping T : $V_3(R)$ $V_3(R)$ is defined by T(x,y,z) = (x-y,x-z).

Show that T is a linear transformation.

5. Let $T: R^3 \rightarrow R^2$ and $H: R^3 \rightarrow R^2$ be defined by T (x, y, z)= (3x, y+z) and H (x, y, z)= (2x-z, y). Compute i) T+H ii) 4T-5H iii) TH iv) HT.

6. If the matrix A is non-singular, show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.

7. State and prove parallelogram law in an inner product space V(F).

8. Prove that the set S = $\left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right), \left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right)\right\}$ is an orthonormal set in the inner product space $R^3(R)$ with the standard inner product.

SECTION - B

Answer ALL the questions. Each question carries TEN marks. 5 X 10 M = 50 M

9(a)) Define vector space. Let V (F) be a vector space. Let W be a non empty sub set of V. Prove that the necessary and sufficient condition for W to be a subspace of V is $\forall\ a, b \in F$ and $\alpha, \beta \in V \Rightarrow a\alpha + b\beta \epsilon W$.

(OR)

(b) Prove that the four vectors (1,0,0), (0,1,0), (0,0,1) and (1,1,1) of $V_3(C)$ form linearly dependent set, but any three of them are linearly independent.

10(a)Define dimension of a finite dimensional vector space. If W is a subspace of a finite dimensional vector space V(F) then prove that W is finite dimensional and dim W $\leq n$.

(OR)

(b) If W be a subspace of a finite dimensional vector space V(F) then Prove that

$$\dim {}^{V}\!/_{W} = \dim V - \dim W.$$

11(a) Find T (x, y, z) where $T: R^3 \to R$ is defined by T (1, 1, 1) =3, T (0, 1, -2) =1, T (0, 0, 1) = -2

(OR)

(b) State and prove Rank Nullity theorem.

12(a) Find the eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} .$$

(OR)

(b) State and prove Cayley-Hamilton theorem.

13(a) State and prove Schwarz's inequality in an Inner product space V(F).

(OR)

(b) Given $\{(2,1,3),(1,2,3),(1,1,1)\}$ is a basis of R^3(R).Construct an ortho normal basis using Gram-Schmidt ortho gonalisation process.