B.A./B.Sc. SECOND YEAR MATHEMATICS SYLLABUS PAPER - III

SEMESTER – III - W.E.F.2016-17

ABSTRACT ALGEBRA

UNIT – 1 : (10 Hrs) GROUPS : -

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group. Composition tables with examples.

UNIT – 2 : (14 Hrs) SUBGROUPS : -

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition – examples-criterion for a complex to be a subgroups.

Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups.

Co-sets and Lagrange's Theorem :-

Cosets Definition – properties of Cosets–Index of a subgroups of a finite groups–Lagrange's Theorem.

UNIT –3 : (12 Hrs) NORMAL SUBGROUPS : -

Definition of normal subgroup – proper and improper normal subgroup–Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group – simple group – quotient group – criteria for the existence of a quotient group.

UNIT – 4 : (10 Hrs) HOMOMORPHISM : -

Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – aultomorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

UNIT – 5 : (14 Hrs) PERMUTATIONS AND CYCLIC GROUPS : -

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups :-

Definition of cyclic group – elementary properties – classification of cyclic groups.

Prescribed Text Book :

A. First course in Abstract Algebra, by J.B. Fraleigh Published by Narosa Publishing house.

Chapters : 1 to 7 and 11 to 13.

Reference Books :

1. A text book of Mathematics for B.A. / B.S. by B.V.S.S. SARMA and others Published by S.Chand & Company New Delhi.
2. Modern Algebra by M.L. Khanna.

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S.V.UNIVERSITY, MODEL PAPER.

THREE YEAR B.A/B.Sc DEGREE EXAMINATIONS.

CHOICE BASED CREDIT SYSTEM

III SEMESTER

PART II : MATHEMATICS

Paper III :ABSTRACT ALGEBRA

(New Syllabus w.e.f 2015-16)

Time: 3 hours **Max Marks :75**

SECTION - A

Answer any FIVE of the following questions. Each question carries 5 marks (5X5 = 25).

1. Show that the fourth roots of unity is an abelian group w.r.t multiplication.
2. Prove that identity element in a group is unique.
3. If Z is the additive group of integers, then prove that the set of all multiples of integers by a fixed number "m" is subgroup of Z.
4. Prove that intersection of two sub groups $H_1$and H_2 of group G,is a subgroup of G.
5. Show that H ={ 1,-1} is a normal subgroup of the group of non-zero real numbers under multiplication.
6. If G is a group of non-zero real numbers under multiplication the prove that $f(x) = x^2 : G \to G$ is a homomorphism. Determine Ker f.
7. Examine whether the following permutation is even or odd.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 9 & 8 \end{pmatrix}$$

8. Define cyclic group and give an example.

(P.T.O)

SECTION - B

Answer ALL of the five questions. Each question carries 10marks (5X10 = 50).

9 a.Prove that the set-Z of all integers form an abelian group w.r.t the operations defined by

$a * b = a + b + 2$,$\forall$ a,b $\in$ Z.

OR

b. Show that the set G= { 1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t X_7 .

10a. Prove that the necessary and sufficient condition for a complex H of a finite group G to be a subgroup is $\forall a, b \in H \Rightarrow a b \in H$.

OR

b. State and prove Lagrange's theorem.

11a. A subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

OR

b. Prove that a sub group H of a group G is normal subgroup of G iff each right coset of H in G is left coset of H in G.

12a. State and prove Fundamental theorem on homomorphism of groups.

OR

b. If $\emptyset : Z_{10} \rightarrow Z_{10}$ be a homomorphism defined by $\emptyset(1) = 8$,then find Ker $\emptyset$.

13a. State and prove Cayley's theorem.

OR

b. The order of a cyclic group is equal to the order of its generator.

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