

**B.A./B.Sc. SECOND YEAR MATHEMATICS SYLLABUS PAPER - III**  
**SEMESTER – III - W.E.F.2016-17**  
**ABSTRACT ALGEBRA**

**UNIT – 1 : (10 Hrs) GROUPS :-**

Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group. Composition tables with examples.

**UNIT – 2 : (14 Hrs) SUBGROUPS :-**

Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition – examples-criterion for a complex to be a subgroups.

Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups.

**Co-sets and Lagrange's Theorem :-**

Cosets Definition – properties of Cosets-Index of a subgroups of a finite groups-Lagrange's Theorem.

**UNIT – 3 : (12 Hrs) NORMAL SUBGROUPS :-**

Definition of normal subgroup – proper and improper normal subgroup-Hamilton group – criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups – Sub group of index 2 is a normal sub group – simple group – quotient group – criteria for the existence of a quotient group.

**UNIT – 4 : (10 Hrs) HOMOMORPHISM :-**

Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties-kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

**UNIT – 5 : (14 Hrs) PERMUTATIONS AND CYCLIC GROUPS :-**

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

**Cyclic Groups :-**

Definition of cyclic group – elementary properties – classification of cyclic groups.

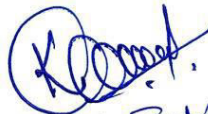
**Prescribed Text Book :**

A. First course in Abstract Algebra, by J.B. Fraleigh Published by Narosa Publishing house.

**Chapters :** 1 to 7 and 11 to 13.

**Reference Books :**

1. A text book of Mathematics for B.A. / B.S. by B.V.S.S. SARMA and others Published by S.Chand & Company New Delhi.
2. Modern Algebra by M.L. Khanna.

  
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**S.V.UNIVERSITY, MODEL PAPER.**

THREE YEAR B.A./B.Sc DEGREE EXAMINATIONS.

CHOICE BASED CREDIT SYSTEM

III SEMESTER

PART II : MATHEMATICS

Paper III : ABSTRACT ALGEBRA

(New Syllabus w.e.f 2015-16)

Time: 3 hours

Max Marks :75

SECTION - A

Answer any FIVE of the following questions. Each question carries 5 marks (5X5 = 25).

1. Show that the fourth roots of unity is an abelian group w.r.t multiplication.
2. Prove that identity element in a group is unique.
3. If  $Z$  is the additive group of integers, then prove that the set of all multiples of integers by a fixed number "m" is subgroup of  $Z$ .
4. Prove that intersection of two sub groups  $H_1$  and  $H_2$  of group  $G$ , is a subgroup of  $G$ .
5. Show that  $H = \{1, -1\}$  is a normal subgroup of the group of non-zero real numbers under multiplication.
6. If  $G$  is a group of non-zero real numbers under multiplication the prove that  $f(x) = x^2 : G \rightarrow G$  is a homomorphism. Determine  $\text{Ker } f$ .
7. Examine whether the following permutation is even or odd.  
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 1 & 4 & 3 & 2 & 5 & 7 & 9 & 8 \end{pmatrix}$$
8. Define cyclic group and give an example.

(P.T.O)

SECTION - B

Answer ALL of the five questions. Each question carries 10marks (5X10 = 50).

9 a. Prove that the set-Z of all integers form an abelian group w.r.t the operations defined by

$$a * b = a + b + 2, \forall a, b \in \mathbb{Z}.$$

OR

b. Show that the set  $G = \{1, 2, 3, 4, 5, 6\}$  is a finite abelian group of order 6 w.r.t  $X_7$ .

10a. Prove that the necessary and sufficient condition for a complex H of a finite group G to be a subgroup is  $\forall a, b \in H \Rightarrow a b \in H$ .

OR

b. State and prove Lagrange's theorem.

11a. A subgroup H of a group G is a normal subgroup of G iff the product of two right cosets of H in G is again a right coset of H in G.

OR

b. Prove that a sub group H of a group G is normal subgroup of G iff each right coset of H in G is left coset of H in G.

12a. State and prove Fundamental theorem on homomorphism of groups.

OR

b. If  $\phi : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_{10}$  be a homomorphism defined by  $\phi(1) = 8$ , then find  $\text{Ker } \phi$ .

13a. State and prove Cayley's theorem.

OR

b. The order of a cyclic group is equal to the order of its generator.

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